THE UNITED REPUBLIC OF TANZANIA

MINISTRY OF EDUCATION, SCIENCE AND TECHNOLOGY

NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

STUDENTS' ITEM RESPONSE ANALYSIS REPORT ON THE FORM TWO NATIONAL ASSESSMENT (FTNA) 2023

BASIC MATHEMATICS

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041 BASIC MATHEMATICS

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FOREWORD

The National Examinations Council of Tanzania is pleased to issue this report on the Students' Item Response Analysis (SIRA) of the Form Two Basic Mathematics Assessment which was held in 2023. The report aims to provide feedback to teachers, students and other education stakeholders regarding the findings of the analysis of the students' performance in Basic Mathematics. The Form Two National Assessment (FTNA) aims to assess students' competence gained after two years of their secondary education and provide feedback to stakeholders with a view to improve teaching and learning.

This report will help students to identify their strengths and weaknesses for the sake of improving their learning before sitting for National Assessment. On the other hand, it will help teachers to identify the challenging areas and take appropriate measures during teaching and learning.

According to the analysis, the students' performance in Basic Mathematics was generally weak. The weak performance was attributed to several factors including inability to: identify multiples of numbers, perform mathematical computations, formulate mathematical statements or expressions and equations from word problems and figures, apply appropriate laws, theorems, postulates, formulae and concepts in solving problems, interpret figures mathematically, draw graphs and approximate numbers.

The National Examinations Council of Tanzania believes that the feedback provided in this report will be useful for teacher, students and all other education stakeholders in taking appropriate measures to improve the performance in future assessments.

Lastly, the Examinations Council appreciates the contribution of the examination officers, examiners and all others who participated in the preparation of this report.

Dr. Said Ally Mohamed
EXECUTIVE SECRETARY

1.0 INTRODUCTION

This report provides both statistical and descriptive analyses of performance of the students who sat for the Form Two National Assessment (FTNA) 2023 in Basic Mathematics. The statistical analysis was based on students' performance on each question. The descriptive analysis was based on reasons for students' success and failure to respond to each question.

The analysis of the subject performance shows that the number of students who sat for the assessment (FTNA) in 2023 was 695,145 out of whom 146,362 students, equivalent to 21.08 per cent, passed. In FTNA 2022, a total of 634,775 students sat for the assessment, out of whom 104,841, that was equivalent to 16.56 per cent passed. Comparatively, the performance has increased by 4.52 per cent.

The assessment paper consisted of 10 compulsory questions, each carrying 10 marks. The performance on each question was rated *Good*, *Average* or *Weak* under the condition that the percentage of students who passed falls in the intervals: 65 –100, 30 – 64 or 0 – 29, respectively. The report also contains some recommendations that are useful for students and teachers to improve the performance in future assessments.

2.0 ANALYSIS OF THE STUDENTS' PERFORMANCE ON EACH QUESTION

This section describes the analysis of students' performance on each question. The analysis includes statistical analysis based on students' performance, and statistical figures showing the summary of students' performance based on the score intervals: 6.5 – 10.0, 3.0 – 6.0 and 0.0 – 2.5 which were rated *Good performance*, *Average performance*, and *Weak performance*, respectively. Three colours, that is *green*, *yellow* and *red* are used to mean good performance, average performance and weak performance, respectively. The section also provides a descriptive analysis based on reasons for the students' success and failure to respond to each question. The analysis includes the provision of the samples of correct and incorrect responses to support the reasons for the students' success or failure to respond to each question.

2.1 Question 1: Numbers and Approximations

This question consisted of two parts, (a) and (b). In part (a), the students were assessed whether or not they were able to list the first twelve multiples of 4 and 5 and then identify the common multiples. In part (b), they were required to evaluate the expression $\frac{2}{25}\times 0.737$ correct to; (i) one significant figure and (ii) three decimal places.

The analysis shows that a total of 695,138 students attempted the question and among them, 540,865 (77.8%) students scored from 0 to 2.5 marks showing that the students' performance on this question was weak. It was further noted that a total of 393,508 (56.6%) students scored zero whereas only 25,147 (3.6%) scored full marks. Figure 1 shows the summary of the students' performance on question 1.

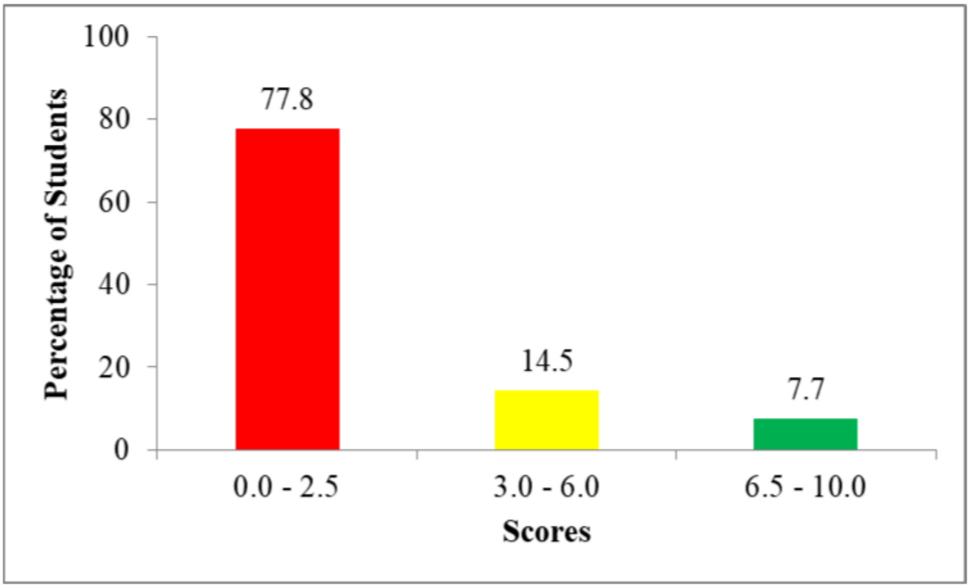


Figure 1: *Students' performance on question 1*

The analysis of students' responses shows that, the weak performance on this question was due to lack of knowledge and skills on the concepts of multiples of numbers, approximations, division and multiplication of decimals. In part (a), the students were unable to list the first twelve multiples of 4 and 5. Some students listed less or more multiples than the required limit of twelve multiples. Also, some students listed the factors instead of multiples of 4 and 5. Moreover, other students computed the Lowest Common Multiples (LCM) of the given numbers instead of listing the common multiples from the list of multiples. In part (b), they performed incorrect multiplication. Most of the students multiplied the numerators incorrectly and failed to divide the product by 25, that is, $\frac{2}{25}\times 0.737=\frac{147}{25}=5.88$ instead of $\frac{2}{25}\times 0.737=\frac{1.475}{25}=0.05896$ which is correct. Some students lacked competence on the concept of approximations since they multiplied correctly but failed to round off to the required number of significant figures and decimal places. Other students failed to understand the requirement of the question. They computed the LCM of 1, 4, 5 and 12 instead of listing the first twelve common multiples of 4 and 5. Extract 1.1 shows a sample of responses of a student who answered this question incorrectly.

1. (a) List the first twelve multiples of 4 and 5 and hence identify the common multiples.

2	12	4	5
2	6	2	5
3	3	1	5
5	1	1	5
	1	1	1

$2 \times 2 \times 3 \times 5 = 60$

Common multiples = 60

(b) Evaluate $\frac{2}{25} \times 0.737$ correct to;

(i) one significant figure.

(ii) three decimal places.

Soln

i/ $\frac{2}{25} \times 0.737$

$$\begin{array}{r} 0.737 \\ \times \quad 2 \\ \hline 1.474 \\ \hline \end{array}$$

$\frac{1.474}{25} = 5.88 = 0.5$

$\therefore = (1sf)$

ii/ $\frac{2}{25} \times 0.737$

$= 5.88$

$\therefore = 0.588$

Extract 1.1: A sample of incorrect responses to question 1

In Extract 1.1, part (a), the student failed to understand the requirement of the question. He/she computed the LCM of 12, 4 and 5 instead of listing the twelve multiples of 4 and 5. In part (b), the student lacked competence on division of decimals by whole numbers.

On the other hand, the analysis of responses shows that the students who performed well on this question were able to apply the knowledge and skills gained after learning the basic concepts of numbers and approximations. In part (a), they correctly listed the first twelve multiples of 4 and 5 and then identified the common multiples, which are 20 and 40. In part (b), the students were able to evaluate $\frac{2}{25} \times 0.737$ correct to; (i) one significant figure and (ii) three decimal places. They multiplied 2 by 0.737 to get 1.474 and then divided the product by 25 to get 0.05896. Those students performed correct approximations of the obtained result. They rounded off 0.05896 to one significant figure as 0.06 and to three decimal places as 0.059. Extract 1.2 shows a sample of correct responses from one of the students who answered this question correctly.

1. (a) List the first twelve multiples of 4 and 5 and hence identify the common multiples.

Multiples of 4 (first twelve) are: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48.
Multiples of 5 (first twelve) are: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60.
Common multiples of 4 and 5 are: 20 and 40.

∴ The common multiples of 4 and 5 are 20 and 40.

(b) Evaluate $\frac{2}{25} \times 0.737$ correct to;

(i) one significant figure.

(ii) three decimal places.

(b) (i) $\frac{2}{25} \times 0.737 = \frac{1.474}{25} = 0.05896 \approx 0.06$

$0.05|896$
$+1$
0.06

∴ $0.05896 \approx 0.06$ correct to one significant figure

$\frac{2}{5} \times 0.737 = 0.058|96$
$+1$
0.059

∴ $0.05896 \approx 0.059$ correct to three decimal places

Extract 1.2: A sample of correct responses to question 1

In Extract 1.2, part (a), the student listed correctly the multiples of 4 and 5 and then identified their common multiples, which are 20 and 40. In part (b), the student computed the value of the expression and rounded off the answer correct to one significant figure and three decimal places.

2.2 Question 2: Fractions, Decimals, and Percentages

This question consisted of parts (a) and (b). In part (a), the students were asked to arrange the fractions $\frac{2}{3}, \frac{4}{7}, \frac{3}{8},$ and $\frac{5}{9}$ in ascending order of magnitude. In (b), the students were required to find the population of Mericho village in 2017, given that in the year 2016 the population was 2800 and in 2017 the population increased by 8%.

The question was attempted by 695,145 students, out of whom 503,226 (72.4%) students scored 0 to 2.5 marks. This indicates that the students' performance on this question was weak. It was further noted that, only 43,917 (6.3%) students scored full marks while a total of 441,159 (63.5%) students scored zero. Figure 2 summarizes the students' performance on question 2.

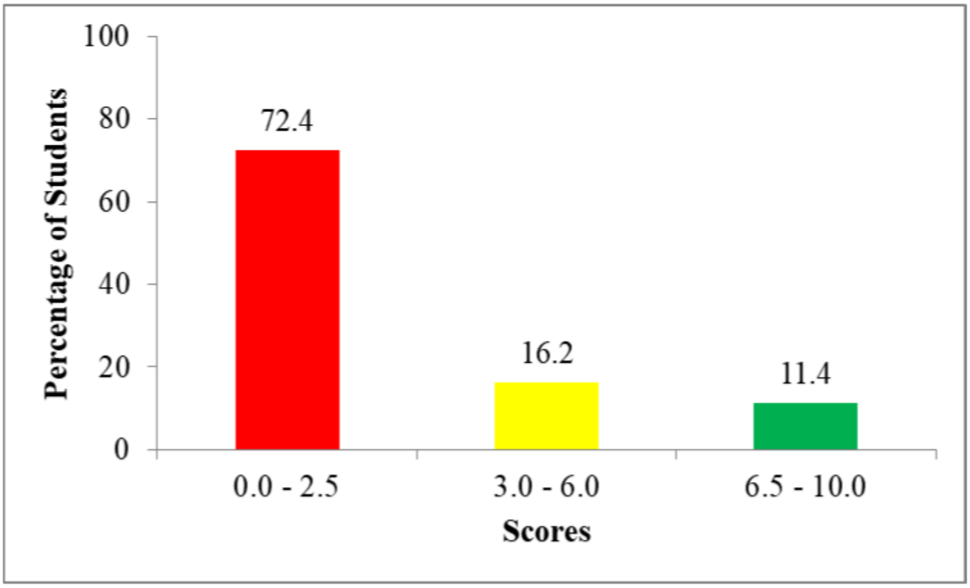


Figure 2: *Students' performance on question 2*

The analysis shows that, the students who performed poorly on this question lacked competence on the basic concepts of fractions, decimals, and percentages. In part (a), they failed to understand the requirement of the question which was to arrange the fractions in ascending order of magnitude. Other students added the given fractions to get $\frac{14}{27}$

$\frac{2}{3}, \frac{3}{8}, \frac{4}{7},$ and $\frac{5}{9}$ as the ascending order of magnitude, which is also an incorrect answer. Majority of the students arranged the fractions in descending order instead of arranging them in ascending order, that is $\frac{2}{3}, \frac{4}{7}, \frac{5}{9},$ and $\frac{3}{8}$. In part (b), the students interpreted wrongly the word problem and extracted wrong data. They responded by multiplying the year 2017 by 8%, that is $8\% \times 2017 = 161.36$, which was incorrect.

Further analysis shows that, some students had the idea of the requirement of the question. They were able to find the increase in population, that is $\frac{8}{100} \times 2800 = 224$, but they failed to add the increase in population to the original population to get the population in 2017. They concluded that the population is 224, which was incorrect. Also, some students managed to find the increase in population, but instead of adding to the original population, they subtracted the increase in population from the original population to get a new population of 2576, that is $2800 - 224 = 2576$ which was incorrect. Furthermore, the analysis shows that some students lacked computation skills since they failed to get the value of $\frac{8}{100} \times 2800$. Extract 2.1 shows a sample of response of a student who answered this question incorrectly.

2. (a) Arrange the given fractions in ascending order of magnitude: $\frac{2}{3}, \frac{4}{7}, \frac{3}{8}$ and $\frac{5}{9}$.

Soln

$\frac{2}{3}, \frac{3}{8}, \frac{4}{7}, \frac{5}{9}$

$\frac{2}{3} + \frac{3}{8} + \frac{4}{7} + \frac{5}{9}$

$= \frac{14}{27}$

(b) In the year 2016 the population of Mericho village was 2,800. In 2017 the population increased by 8%. What was the population in 2017?

Soln

2016 mericho village was 2,800
in 2017 8 %

$$\begin{array}{r} 2800 \\ \times \quad 8 \\ \hline 22800 \end{array}$$

$\therefore$ The population in 2017 is 22,800.

Extract 2.1: A sample of incorrect responses to question 2

In Extract 2.1, part (a) shows that the student did not understand the question and lacked knowledge and skills on operations on fractions. He/she added the numerators as well as the denominators instead of arranging the fractions in ascending order. In part (b), the student ignored the percentage symbol and multiplied the numbers incorrectly.

Despite the weak performance, there were students who performed well on this question. The students had appropriate competence on fractions, decimals, and percentages. In part (a), they were able to arrange the fractions $\frac{2}{3}, \frac{4}{7}, \frac{3}{8},$ and $\frac{5}{9}$ in ascending order of magnitude. They determined the LCM of the denominators and multiplied the given fractions by the LCM of their denominators to get the corresponding whole numbers. Some of the students converted the fractions into percentages or decimals. They managed to use the equivalent whole numbers, percentages or decimals to arrange the given fractions in ascending order of magnitude as $\frac{3}{8}, \frac{5}{9}, \frac{4}{7},$ and $\frac{2}{3}$. In part (b), they calculated 8% of the population in 2016, that is $8\% \times 2800 = 224$. They were able to recognize that, the population in 2017 is the sum of the population in 2016 and its 8% increase. Thus, they added the population in 2016 and its 8% to get 3024, which was the correct answer. Extract 2.2 shows a sample of responses of a student who answered this question correctly.

2. (a) Arrange the given fractions in ascending order of magnitude: $\frac{2}{3}, \frac{4}{7}, \frac{3}{8}$ and $\frac{5}{9}$.

Solution

Changing the fractions into Percentages

$\frac{2}{3} \times 100\% = 66.67\%$

$\frac{4}{7} \times 100\% = 57.14\%$

$\frac{3}{8} \times 100\% = 37.5\%$

$\frac{5}{9} \times 100\% = 55.56\%$

Percentages in ascending order

$= 37.5\%, 55.56\%, 57.14\%, 66.67\%$

∴ The fractions in ascending order is $\frac{3}{8}, \frac{5}{9}, \frac{4}{7}, \frac{2}{3}$

(b) In the year 2016 the population of Mericho village was 2,800. In 2017 the population increased by 8%. What was the population in 2017?

Solution

Given, In Mericho village

Population in 2016 = 2,800.

Percentage increase of population in 2017 = 8%

Now, formula for percentage increase in population for Mericho village.

$\% \text{ Increase} = \frac{\text{Increased number of 2017}}{\text{Population in 2016}} \times 100\%$

$8\% = \frac{\text{Increase}}{2800} \times 100\%$

$\frac{2800 \times 8}{100} = \frac{100(\text{Increase})}{100}$

Increased people = 224.

Thus, 224 people increased more in 2017.

Now, for total population in 2017

Total population = Former population (of 2016) + Increase in people.

Total population in 2017 = 2800 + 224

= 3024

∴ The population of Mericho village in 2017 is 3024

Extract 2.2: A sample of correct responses to question 2

In Extract 2.2, part (a), the student converted all the fractions into percentages by multiplying them by 100% correctly. He/she arranged the obtained percentages in ascending order as well as their corresponding

$\frac{3}{8}, \frac{5}{9}, \frac{4}{7}$, and $\frac{2}{3}$. In part (b), the student calculated 8% of 2800 to get a population of 224. He/she summed up the population in 2016 and its 8% to obtain 3024, which was the population in 2017.

2.3 Question 3: Units, Ratios, Profit and Loss

This question comprised two parts, (a) and (b). In part (a), students were required to determine the number of kilograms each school got, if 1,000 tonnes of maize were shared equally among 25 schools. In part (b), students were asked to find the profit and selling price, if a shopkeeper bought a radio for sh. 80,000 and sold it at a profit of 20%.

The analysis shows that a total of 695,142 students attempted the question and among them, 509,545 (73.3%) students scored 2.5 marks or less. Furthermore, 376,075 (54.1%) students scored zero on this question showing that the general performance was weak. However, there were 52,401 (7.5%) students who scored full marks. Figure 3 shows the students' performance on question 3.

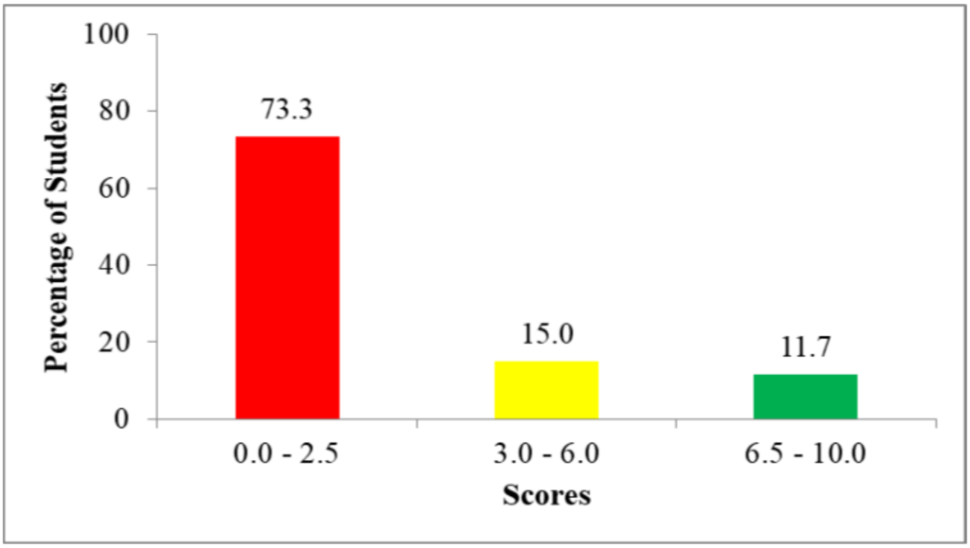


Figure 3: *Students' performance on question 3*

The analysis of students' responses indicates that students who had weak performance on this question lacked competence on the concept of metric units. Also, they had inadequate knowledge and skills of solving real life problems on ratios, profit and loss. In part (a), students failed to divide 1,000 tonnes by 25, instead they multiplied 1,000 by 25 to get 25,000 kilograms. This misconception resulted from failure to understand the term "shared equally" as applied in real life situations. Furthermore, there were students who divided 1,000 tonnes by 25 but got incorrect answers due to lack of skills of performing division of whole numbers. Some students related wrongly the metric units of mass as $1 \text{ tonne} = 10 \text{ kg}$ or $10 \text{ tonnes} = 1 \text{ kg}$ which propagated to an incorrect answer. This indicates that the students lacked competence on the conversion of metric units.

In part (b), most of the students applied incorrect formulae for finding profit. They wrote $\text{profit} = \frac{\text{buying price}}{\text{percentage profit}}$, $I = \frac{PRT}{100}$, where I is the simple interest on the principal (P) for the time (T) or $\text{profit} = \text{buying price} - \%\text{ profit}$ or $\text{profit} = \text{buying price} - \%\text{ profit}$. Thus, all the arithmetics performed were incorrect. Extract 3.1 shows a sample of responses of a student who answered this question incorrectly.

3. (a) If 1,000 tonnes of maize were shared equally among 25 schools, how many kilograms did each school get?

Soln

tonnes 1,000

shared equally 25

tonnes 1,000

+ 25

25000

∴ the Kilograms did each school get is 25,000

(b) A shopkeeper bought a radio for sh. 80,000 and sold it at a profit of 20%. What were the profit and selling price?

Soln

~~bouth bought radio sh 80,000~~

$I = \frac{PRT}{100}$

80,000 20% = 2

1000

~~80,000~~ 2

= 80x2

= 160

∴ the selling rice is 160

Extract 3.1: A sample of incorrect responses to question 3

Extract 3.1 shows that, in part (a), the student multiplied the given numbers contrary to the requirement of the question. In part (b), the student applied a wrong formula in computing the selling price.

On the other hand, the analysis of responses shows that the students who performed well on this question demonstrated adequate knowledge on metric units, ratios, profit and loss. In part (a), they were able to divide

$\text{percentage profit} = \frac{\text{profit made}}{\text{buying price}} \times 100\%$. Those students extracted properly the useful data from the word problem, that is $\text{Buying price} = 80,000$ and $\text{Percentage profit} = 20\%$. They substituted in the formula and applied their arithmetic skills to compute the profit. That is, $\text{Profit made} = 16,000$. Finally, they applied the formula Profit = Selling price – Buying price to compute the selling price. That is, Selling price = Buying price + Profit made = sh 80,000 + sh 16,000. They correctly obtained the selling price of sh 96,000. Extract 3.2 shows a sample of correct responses from one of the students who answered this question correctly.

3. (a) If 1,000 tonnes of maize were shared equally among 25 schools, how many kilograms did each school get?

soln

No of maize = 1000 t.
Total schools = 25.
No of kilograms that each school got.
= 1000t / 25
= 40 tonnes.
change 40t → kg.
1t = 1000kg
40t = ?
= 40t × 1000kg / 1t
= 40,000 kg.

∴ Each school got 40,000 kg.

(b) A shopkeeper bought a radio for sh. 80,000 and sold it at a profit of 20%. What were the profit and selling price?

Soln.

Buying price = sh 80,000

% profit = 20%.

selling price = ?

profit = ?

But,

$\% P = \frac{P}{B.P} \times 100\%$.

$20\% = \frac{P}{sh\,80,000} \times 100\%$

$\frac{100P}{100} = \frac{sh\,80,000 \times 20}{100}$.

P = sh 16000

Profit = sh 16,000.

Again,

Profit = Selling price − Buying price

sh 16,000 = S.P − sh 80,000.

S.P = sh 16,000 + sh 80,000

S.P = sh 96,000.

Selling price = sh 96,000.

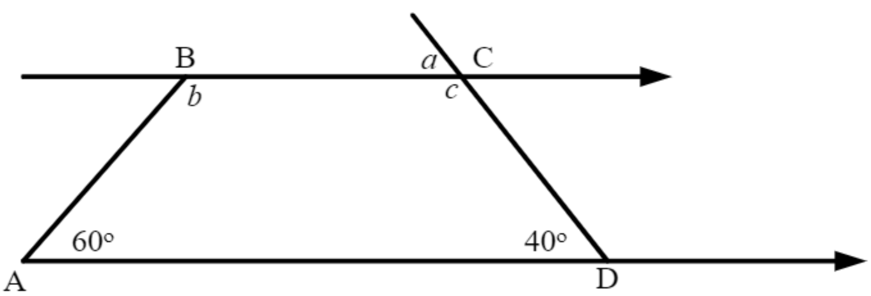
∴ Profit is sh 16,000 and Selling price is sh 96,000.

Extract 3.2: A sample of correct responses to question 3

Extract 3.2 shows that, in part (a), the student correctly divided 1,000 tonnes by 25 and converted the 40 tonnes obtained into kilograms. In part (b), the student managed to find the profit and selling price correctly after applying appropriate formulas.

2.4 Question 4: Geometry, Perimeters and Areas

The question consisted of two parts, (a) and (b). In part (a), students were required to find the values of the angles marked a, b, and c of a trapezium ABCD in the following figure.



In part (b), the students were assessed whether or not they were able to find the perimeter and area of the floor of a square room of length $5\,\text{m}$.

The data analysis reveals that the question was attempted by 695,141 students. Further analysis shows that 535,561 (77.0%) students scored below 3 marks, indicating that the students' performance on this question was weak. It was also noted that, only 46,074 (6.6%) students scored full marks while a total of 419,279 (60.3%) scored zero. Figure 4 summarizes the students' performance on question 4.

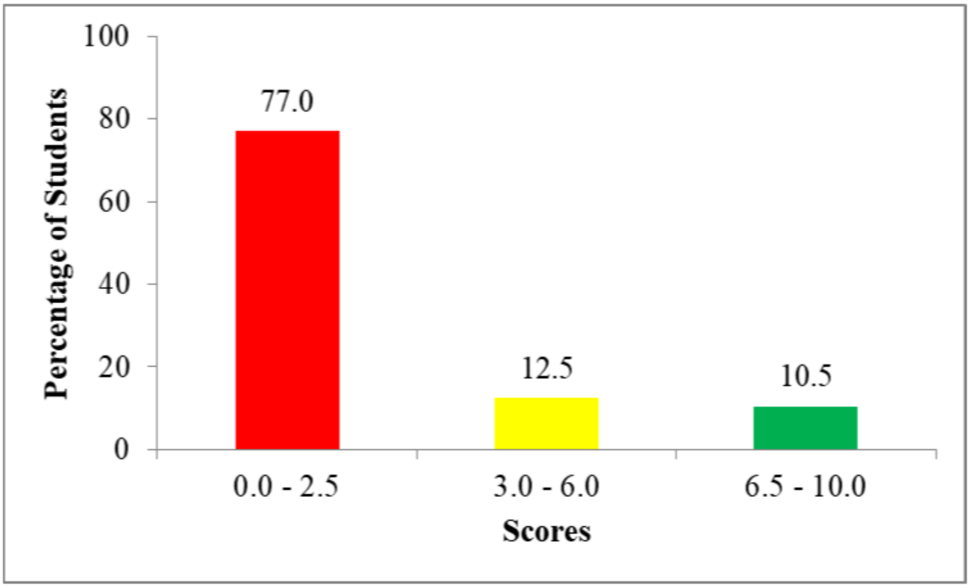
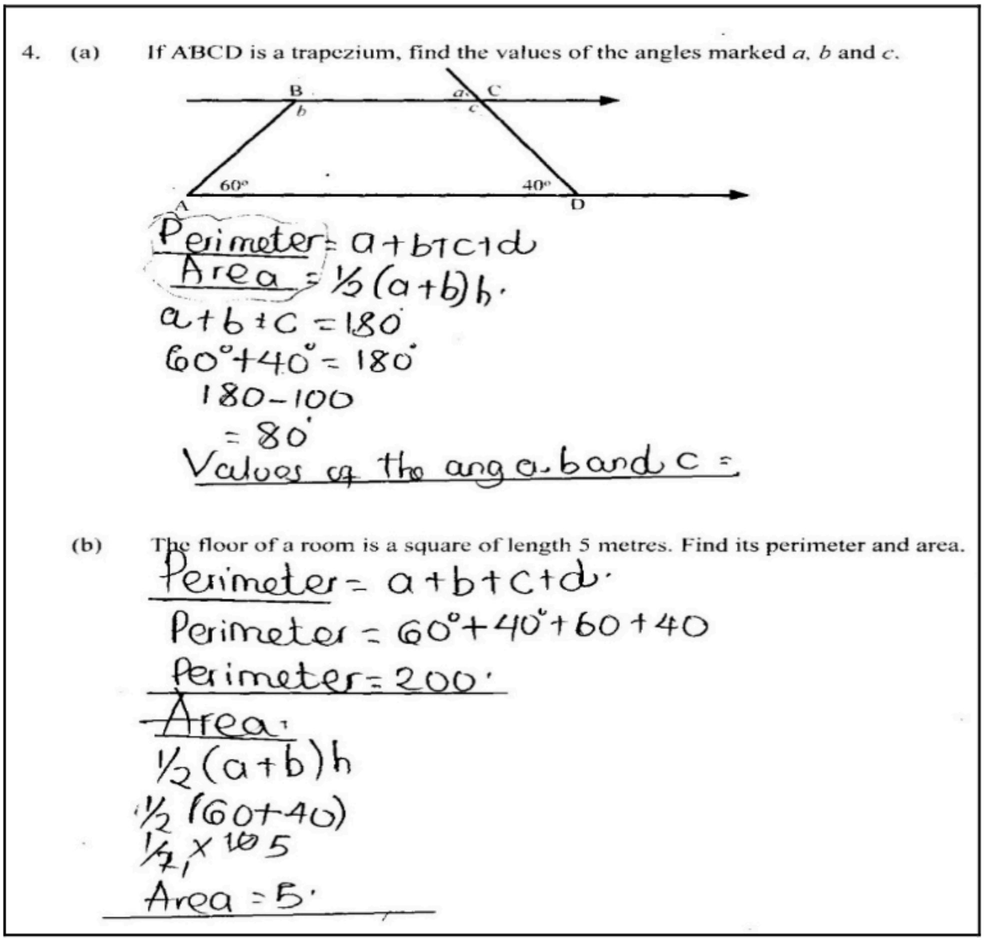


Figure 4: *Students' performance on question 4*

The analysis of students' responses indicates that most of the students who had weak performance on this question were unable to demonstrate the assessed competence on geometry. In part (a), they failed to identify the relationship of angles formed by parallel lines and a transversal. For example, some of the students considered that angles a, b, c are supplementary, that is $a+b+c=180°$, which was incorrect. Furthermore, other students failed to understand the requirement of the question. They applied the formula for finding the area of a trapezium, that is $\text{Area} = \frac{1}{2}(a+b)h$ which was a wrong approach. In part (b), they were unable to find the perimeter and area of a square shaped floor. They failed to apply the correct formulae, instead they applied wrong formulae such as

$\text{Perimeter} = \text{L} \times \text{L} \times \text{L} \times \text{L}$ and $\text{Area} = 4 \times \text{L}$, where L is the length of the square. Extract 4.1 represents a sample of a response from one of the students who failed to answer this question correctly.


4. (a) If ABCD is a trapezium, find the values of the angles marked *a*, *b* and *c*.

Perimeter = a+b+c+d
Area = ½(a+b)h.
a+b+c = 180
60°+40° = 180°
180−100
= 80°
Values of the ang a, b and c =

(b) The floor of a room is a square of length 5 metres. Find its perimeter and area.

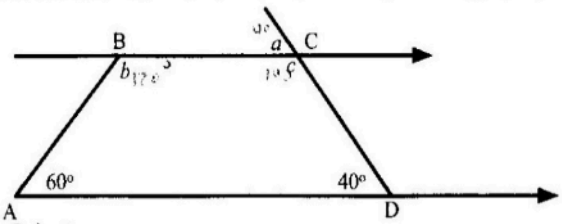
Perimeter = a+b+c+d.
Perimeter = 60°+40°+60+40
Perimeter = 200.
Area.
½(a+b)h
½(60+40)
½ × 10 5
Area = 5.

Extract 4.1: A sample of incorrect responses to question 4

In Extract 4.1, part (a), the student used inappropriate formulae in computing the marked angles. In part (b), the student failed to apply the formulae for perimeter and area of a square. He/she used the numbers 40, 40°, 60 and 60° which were not provided in the equation.

However, there were students who had good performance on this question. In part (a), they scored full marks because they had sufficient knowledge on the basic concepts of geometry. They applied the properties of the quadrilateral ABCD and those of the angles formed by the transversal $\overline{DC}$. They computed the values of the marked angles to obtain $a = 40°$, $b = 120°$, and $c = 140°$. In part (b), the students were able to find the perimeter and area of the floor of a square room of length 5 m by using the formulae $\text{Perimeter} = 4 \times \text{Length}$ and $\text{Area} = \text{Length} \times \text{Length}$. Thus, they obtained a perimeter of 20 m and an area of $25\,\text{m}^2$. This shows that those students were competent on the basic concepts of geometry, perimeters and areas. Extract 4.2 shows a sample of correct response of a student who answered this question correctly.

4. (a) If ABCD is a trapezium, find the values of the angles marked *a*, *b* and *c*.



Solution

$\angle a = C\hat{D}A$ – Alternate interior angles

$\angle a = 40^{\circ}$

$\angle c = 180^{\circ} - \angle a$ – Degree measure in a straight line

$\angle c = 180^{\circ} - 40^{\circ}$

$\angle c = 140^{\circ}$

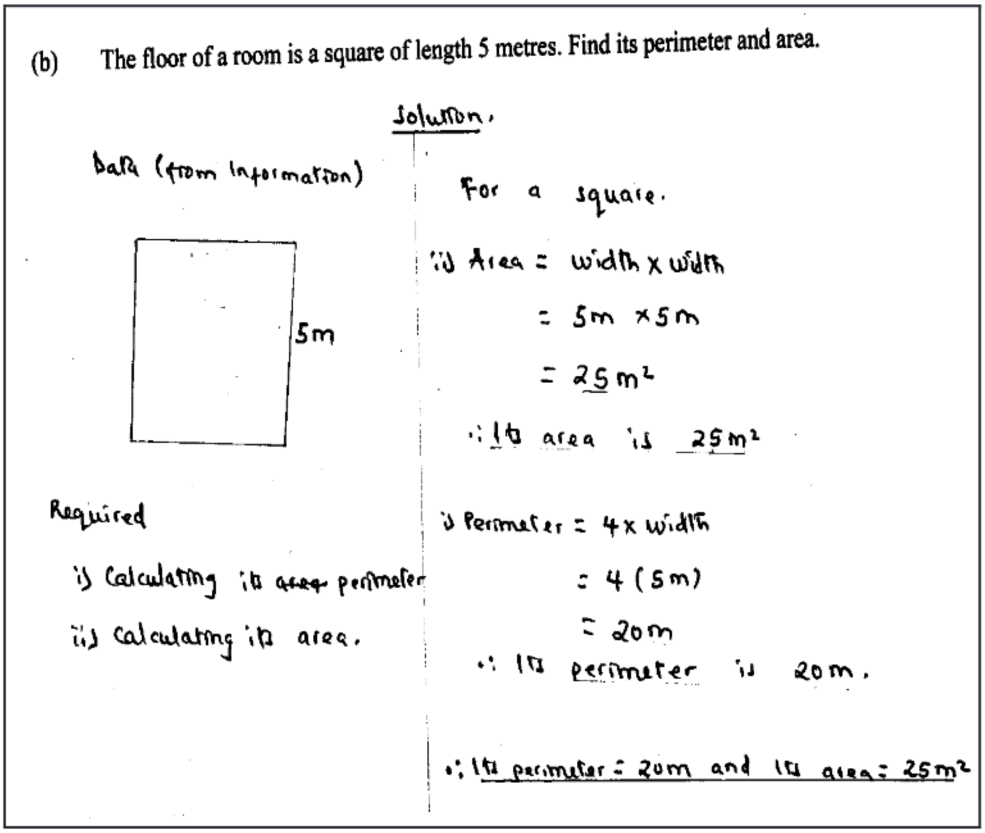
$\angle b = 360^{\circ} - (\angle c + C\hat{D}A + B\hat{A}D)$ – Degree measure in a trapezium

$\angle b = 360^{\circ} - (140^{\circ} + 40^{\circ} + 60^{\circ})$

$\angle b = 360^{\circ} - 240^{\circ}$

$\angle b = 120^{\circ}$

$\therefore$ Angles a, b and c are 40°, 120° and 140° respectively


(b) The floor of a room is a square of length 5 metres. Find its perimeter and area.

Solution.

Data (from information)

5m

Required

i) Calculating its ~~area~~ perimeter

ii) Calculating its area.

For a square.

ii) Area = width x width

= 5m x 5m

= 25 m²

∴ Its area is 25m²

i) Perimeter = 4 x width

= 4 (5m)

= 20m

∴ Its perimeter is 20m.

∴ Its perimeter = 20m and Its area = 25m²

Extract 4.2: A sample of correct responses to question 4

Extract 4.2 shows that, in part (a), the student applied the properties of angles formed by a transversal and the properties of quadrilaterals to obtain the values of the angles marked a, b and c. In part (b), the student used appropriate formulae and performed correct arithmetics to get the perimeter and area of the square shaped floor.

2.5 Question 5: Algebra and Quadratic Equations

This question consisted of two parts, (a) and (b). In part (a), the students were assessed if they were able to apply algebraic operations on an equation with one unknown to determine the value of y,

$\frac{6y+1}{4}=\frac{5(y+5)}{6}$. In part (b), the students were required to solve the equation $3x^2-7x-6=0$ by using the method of completing the square.

The analysis shows that the question was attempted by a total of 695,137 students and among them 598,492 (86.1%) scored 0 to 2.5 marks indicating that the students' performance on this question was weak. Further analysis reveals that 527,296 (75.9%) students scored zero whereas 10,670 (1.5%) students scored all 10 marks. The summary of the students' performance on question 5 is shown in Figure 5.

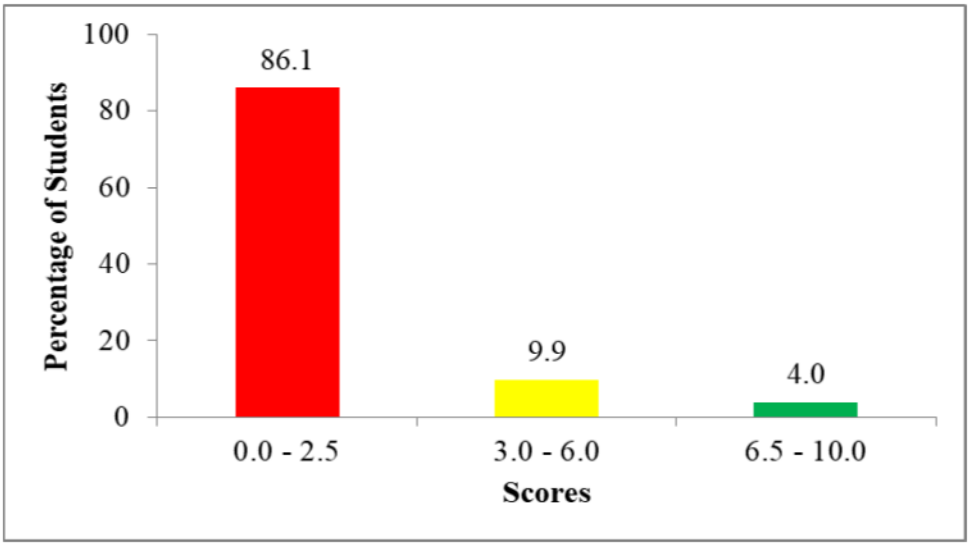


Figure 5: *Students' performance on question 5*

The analysis of responses shows that the students who performed poorly on this question lacked knowledge and skills on algebra and quadratic equations. In part (a), most of the students failed to obtain the value of y

$\frac{6y+1}{4} = \frac{5(y+5)}{6}$ as $\frac{6y+1}{4} = \frac{5y+5}{6}$ instead of $\frac{6y+1}{4} = \frac{5y+25}{6}$. Also, they failed to rearrange the equation in linear form with integer coefficients.

In part (b), majority of the students failed to develop an equivalent equation that reflects a perfect square whose coefficient of x^2 is 1. This led to difficulties in calculations and ultimately wrong answers. Furthermore, some students managed to develop the equivalent equation with 1 as the coefficient of x^2 but failed to transform it into a perfect square. Other students were able to transform the given equation into a perfect square but made wrong calculations when solving for the values of x. The analysis also reveals that some students attempted the question by using the method of factorization which was contrary to the requirement. This suggests that they were not competent in using the method of completing the square. Extract 5.1 shows a sample of responses from a student who failed to answer the question correctly.

5. (a) If $\frac{6y+1}{4} = \frac{5(y+5)}{6}$, find the value of y correct to three significant figures.

Soln

$\frac{6y+1}{4} = \frac{5(y+5)}{6}$

$= \frac{6y+1}{4} = \frac{5y+5}{6}$

$\frac{6y+5y}{4} = \frac{5-1}{6}$

$\frac{11y}{4} = \frac{4}{6}$

$= \frac{44y}{4} = \frac{24}{6}$

$= 11y = 4$

(b) Solve the equation $3x^2 - 7x - 6 = 0$ by completing the square.

Soln

$3x^2 - 7x - 6 = 0$

$9x - 7x - 6 = 0$

$2x - 6 = 0$

$2x = 0 - 6$

$= 2x = 0 - 6$

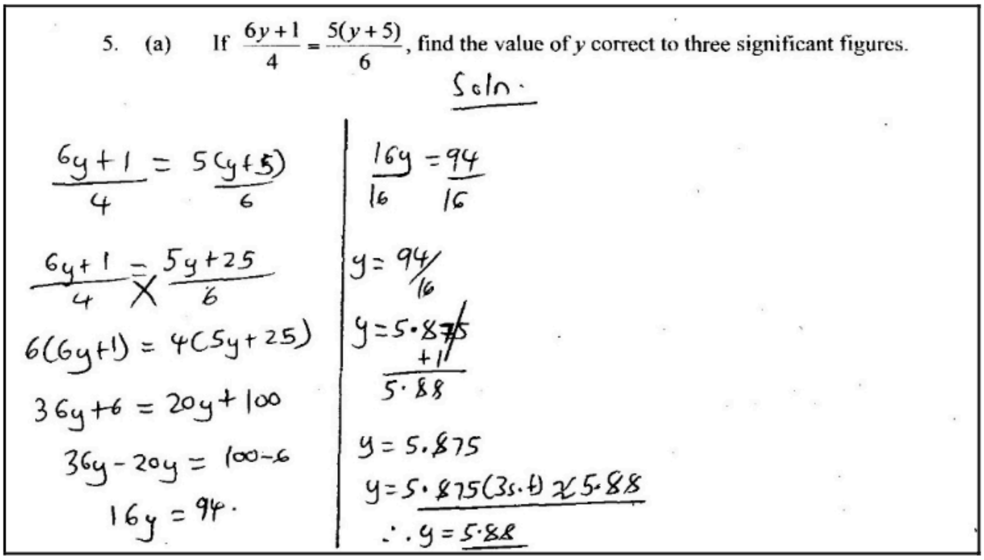
Extract 5.1: A sample of incorrect responses to question 5

In Extract 5.1, part (a), the student failed to open the brackets in the equation and was unable to perform cross-multiplication. In part (b), the student failed to transform the given quadratic equation into a perfect square. He/she changed the term $3x^2$ to $9x$, which was a wrong procedure.

Despite the weak performance, there were students who performed well on this question. The students demonstrated sufficient competence in solving

y correct to three significant figures in the equation $\frac{6y+1}{4}=\frac{5(y+5)}{6}$. They were able to apply suitable algebraic operations to get $y=5.875$ and then round off the answer to three significant figures to obtain $y=5.88$ as required.

In part (b), the students were able to transform the quadratic equation $3x^2-7x-6=0$ into its equivalent equation whose coefficient of x^2 is 1. From the equivalent equation, they established a quadratic equation which reflects a perfect square and solved for the values of x to get $x=3$ or $x=-\frac{2}{3}$. Extract 5.2 represents a sample of correct responses from one of the students who answered this question correctly.



5. (a) If $\frac{6y+1}{4}=\frac{5(y+5)}{6}$, find the value of y correct to three significant figures.

Soln.

$\frac{6y+1}{4}=\frac{5(y+5)}{6}$

$\frac{6y+1}{4}=\frac{5y+25}{6}$

$6(6y+1)=4(5y+25)$

$36y+6=20y+100$

$36y-20y=100-6$

$16y=94.$

$\frac{16y}{16}=\frac{94}{16}$

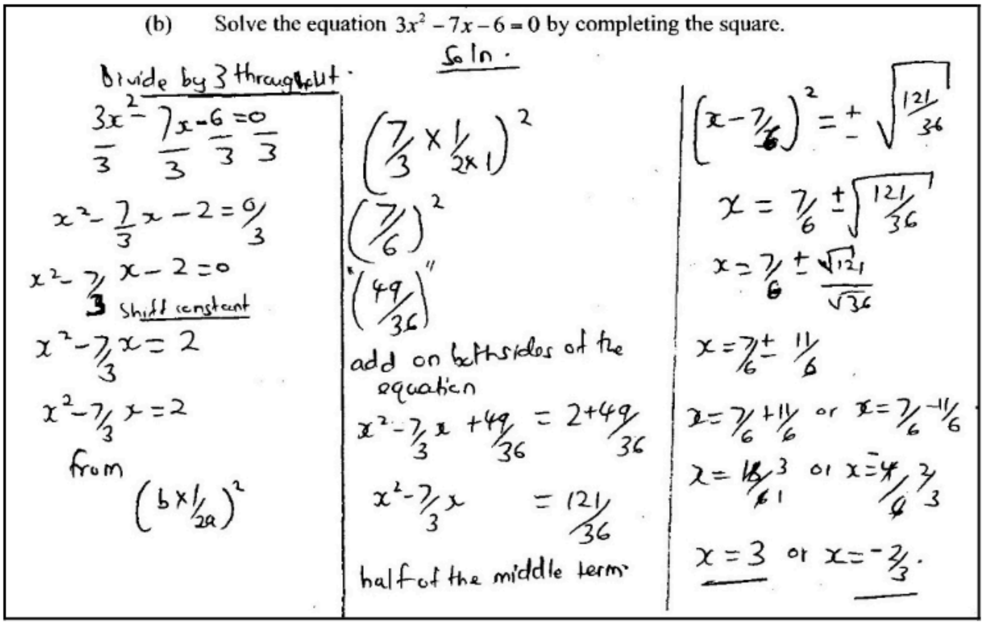
$y=94/16$

$y=5.875$

$y=5.875$

$y=5.875 (3s.f) \approx 5.88$

$\therefore y=5.88$


(b) Solve the equation $3x^2 - 7x - 6 = 0$ by completing the square.

Soln.

Divide by 3 throughout.

$\frac{3x^2}{3} - \frac{7x}{3} - \frac{6}{3} = \frac{0}{3}$

$x^2 - \frac{7}{3}x - 2 = \frac{0}{3}$

$x^2 - \frac{7}{3}x - 2 = 0$

Shift constant

$x^2 - \frac{7}{3}x = 2$

$x^2 - \frac{7}{3}x = 2$

from

$\left(b \times \frac{1}{2a}\right)^2$

$\left(\frac{7}{3} \times \frac{1}{2 \times 1}\right)^2$

$\left(\frac{7}{6}\right)^2$

$\left(\frac{49}{36}\right)$

add on both sides of the equation

$x^2 - \frac{7}{3}x + \frac{49}{36} = 2 + \frac{49}{36}$

$x^2 - \frac{7}{3}x \quad = \frac{121}{36}$

half of the middle term.

$\left(x - \frac{7}{6}\right)^2 = \pm\sqrt{\frac{121}{36}}$

$x = \frac{7}{6} \pm \sqrt{\frac{121}{36}}$

$x = \frac{7}{6} \pm \frac{\sqrt{121}}{\sqrt{36}}$

$x = \frac{7}{6} \pm \frac{11}{6}$

$x = \frac{7}{6} + \frac{11}{6}$ or $x = \frac{7}{6} - \frac{11}{6}$

$x = \frac{18}{6}$ or $x = -\frac{4}{6}$

$x = 3$ or $x = -\frac{2}{3}$.

Extract 5.2: A sample of correct responses to question 5

Extract 5.2 shows that, in part (a), the student managed to open the brackets and subsequently cross-multiplied correctly. He/she collected the like terms and performed division and finally approximated the value of y to three significant figures as the question demanded. In part (b), the student developed an equivalent quadratic equation whose coefficient of x^2 is 1 and established the equation reflecting a perfect square to compute the values of x as required.

2.6 Question 6: Coordinate Geometry and Geometrical Transformations

This question comprised two parts, (a) and (b). In part (a), the students were required to find the gradient of a straight line joining the points $(-1, 2)$ and $(3, -5)$. In part (b), the students were required to find the image of the point $P(-3, 7)$ after a reflection in the x-axis and y-axis.

The analysis shows that out of 695,139 students who attempted the question 539,189 (77.6%) students scored 2.5 marks or less and 405,406 (58.3%) scored zero. Based on these scores, the students' performance on this question was weak. On the contrary, 34,743 (5.0%) students scored all the marks allotted to the question. Figure 6 recapitulates the students' performance on question 6.

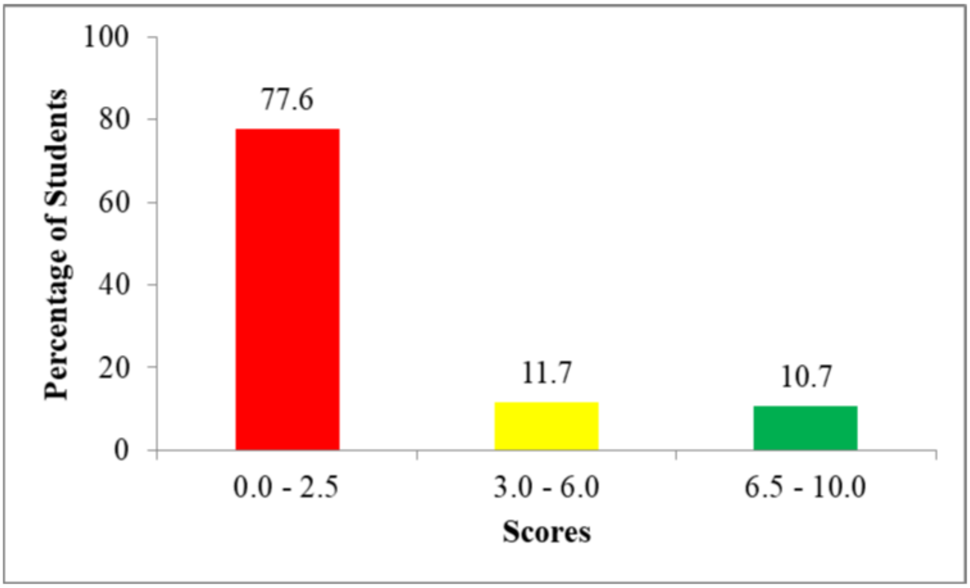


Figure 6: *Students' performance on question 6*

The analysis of the students' responses shows that the students who failed to answer this question correctly lacked knowledge and skills on the basic concepts of coordinate geometry and geometrical transformations. In part (a), the students failed to apply the correct formula for finding the gradient of a line which passes through two given points. They used incorrect formulae such as $\text{gradient} = \dfrac{\text{change in } x}{\text{change in } y} = \dfrac{x_2 - x_1}{y_2 - y_1}$.

x and y values. Some of the students substituted the given points in the formula correctly but were unable to get the correct answer due to lack of knowledge on operations of integers. In part (b), the students were not able to find the images of the given point after a reflection in the x-axis and y-axis. Those students failed to locate the given point on the coordinate plane and hence, they were not able to get the correct images of the given point. This shows that the students lacked competence and skills in determining the image of the given point. Extract 6.1 shows a sample of responses from a student who failed to answer this question correctly.

6. (a) Find the gradient of a straight line joining the points (-1, 2) and (3, -5).

Soln

$$M = \frac{x_2 - x_1}{y_2 - y_1}$$

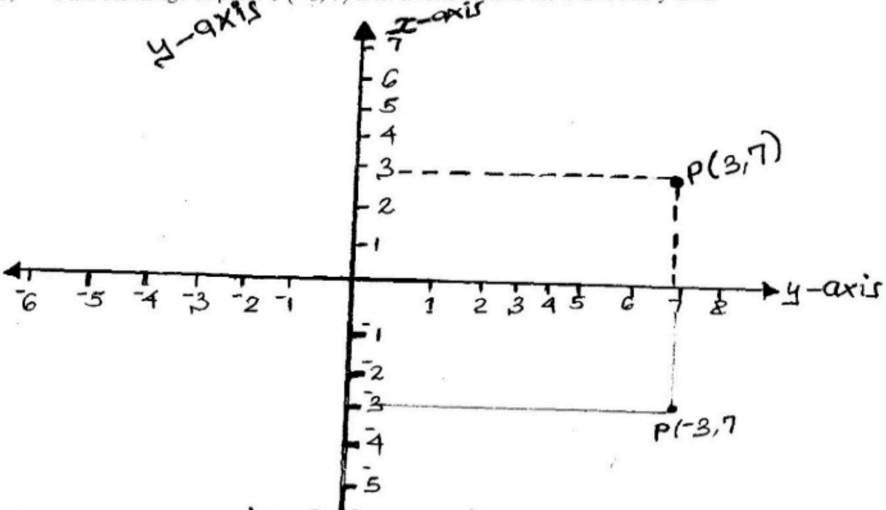
$$M = \frac{3 + {}^{+}1}{{}^{-}5 - 2}$$

$$M = \frac{4}{3}$$

$$M = 1.33$$

∴ The gradient is 1.3

(b) Find the image of point $P(-3, 7)$ after a reflection in the x-axis and y-axis.



∴ After reflection is $P_1(3,7)$ in y-axis

Extract 6.1: A sample of incorrect responses to question 6

In Extract 6.1, part (a), the student used incorrect formula to find the gradient of a straight line joining the points $(-1, 2)$ and $(3, -5)$, that is $m = \frac{x_2 - x_1}{y_2 - y_1}$, which resulted to an incorrect answer. In part (b), the student failed to identify the coordinate axes. Therefore, he/she failed to obtain the correct images of the given point after a reflection on the x and y-axes.

On the other hand, the analysis of responses shows that, the students who performed well in part (a) showed good understanding of the basic concepts of coordinate geometry. They were able to determine the gradient of a straight line joining the points $(-1, 2)$ and $(3, -5)$ correctly. This was achieved by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{y_1 - y_2}{x_1 - x_2}$. They identified the points $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (3, -5)$ and substituted in the formula to get $m = -\frac{7}{4}$, which is the correct gradient of the line passing through the given points. In part (b), the students were able to find the image of the point $\mathrm{P}(-3, 7)$ after a reflection in the x-axis and y-axis. They applied the formulas $\mathrm{M}_x(x, y) = (x, -y)$ and $\mathrm{M}_y(x, y) = (-x, y)$ for reflection on the x and y-axes, respectively. Some students located the point $\mathrm{P}(-3, 7)$ and its images after reflections on x and y-axes in an xy-plane. They correctly obtained the reflection of the point $\mathrm{P}(-3, 7)$ on the x- axis and y- axis as $(-3, -7)$ and $(3, 7)$, respectively. This shows that the students gained sufficient competence on the basic concepts of geometrical transformations. Extract 6.2 shows a sample of correct responses from one of the students who answered this question correctly.

6. (a) Find the gradient of a straight line joining the points (-1, 2) and (3, -5).

Soln.

gradient = $\frac{\Delta y}{\Delta x}$

gradient = $\frac{y_2 - y_1}{x_2 - x_1}$

whereby,

$(x_1, y_1)(x_2, y_2) = (-1, 2)(3, -5)$

gradient = $\frac{-5-2}{3-^-1}$

gradient = $\frac{-7}{3+1}$

gradient = $-\frac{7}{4}$

∴ The gradient of the two points is $\frac{-7}{4}$

(b) Find the image of point $P(-3, 7)$ after a reflection in the x-axis and y-axis.

From, My-axis $(x, y) = (^-x', y')$

My-axis $P(^-3, 7) = P'(3, 7)$

$= P'(3, 7)$

∴ The image of point $P(^-3, 7)$ after a reflection in the y-axis is $P'(3, 7)$

Again,

From, Mx-axis $(x, y) = (x', ^-y')$

Mx-axis $P(^-3, 7) = P'(^-3, ^-7)$

$= P'(^-3, ^-7)$

∴ The image of point $P(^-3, 7)$ after a reflection in the x-axis is $P'(^-3, ^-7)$.

Extract 6.2: A sample of correct responses to question 6

In Extract 6.2, part (a), the student used the correct formula to find the gradient of the line, that is $m = \frac{y_2 - y_1}{x_2 - x_1}$. He/she substituted the values $x_1 = -1$, $y_1 = 2$, $x_2 = 3$, and $y_2 = -5$ in the formula to get $m = -\frac{7}{4}$. In part (b), the student applied the appropriate formulae to obtain the images of the point after a reflection in the x - axis and y - axis.

2.7 Question 7: Exponents, Radicals, and Logarithms

This question consisted of two parts, (a) and (b). Part (a) assessed the students' ability to solve for the value of n in the equation $16^{3-n} \times 2^{(1+n)} = \frac{1}{2}$, leaving the answer in improper fraction form. In part (b), the students were required to find the value of x in the equation $\log(2x^2 + 1) + \log 4 = \log(7x^2 + 8)$.

The data indicate that 695,140 students attempted the question out of whom, 630,163 (90.6%) students scored below 3 marks and 581,545 (83.7%) scored zero. This confirms that the students' performance on this question was weak. The data also indicate that there were 2,298 (0.3%) students who scored full marks. Figure 7 summarizes the students' performance on question 7.

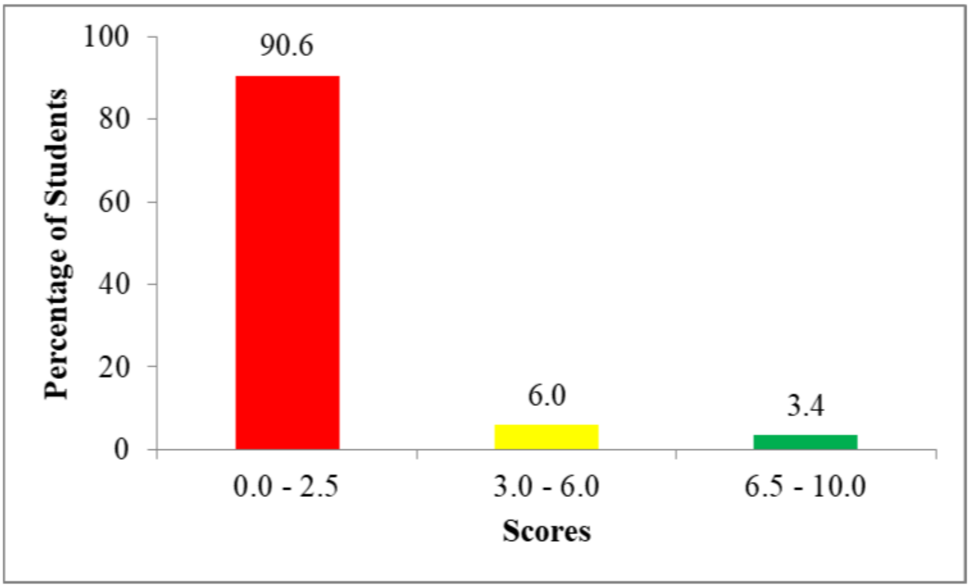


Figure 7: *Students' performance on question 7*

The response analysis shows that, majority of the students failed to answer this question correctly due to lack of competence on the laws of exponents and logarithms. In part (a), the students were not able to apply the reciprocal law of exponents, that is $\frac{1}{a^n} = a^{-n}$,

$16^{(3-n)} \times 2^{(1+n)} = \frac{1}{2}$ as $16^{(3-n)} \times 2^{(1+n)} = {}^{-}2$ instead of $16^{(3-n)} \times 2^{(1+n)} = 2^{-1}$.

$16 = 2 \times 2 \times 2 \times 2 = 2^4$.

$$16^{(3-n)} \times 2^{(1+n)} = (16 \times 2)^{(3-n)(1+n)} = 32^{(3-n)(1+n)}$$

n. Other students multiplied the bases by their exponents, that is $16(3-n) \times 2(1+n) = \frac{1}{2}$

$x^m \times x^n = x^{m+n}$. For example, they wrongly wrote $2^{4(3-n)} \times 2^{(1+n)} = 2^{4(3-n)\times(n+1)}$ instead of $2^{4(3-n)} \times 2^{(1+n)} = 2^{4(3-n)+(n+1)}$. Those students multiplied the exponents instead of adding them. In part (b), the students failed to apply the laws of logarithms to solve the equation $\log(2x^2+1)+\log 4=\log(7x^2+8)$. Most of them wrote the expression $\log(2x^2+1)+\log 4$ incorrectly as $\log(2x^2+1+4)$ instead of $\log[(2x^2+1)\times 4]$ which was the correct step. Furthermore, some of the students re-arranged the equation in the form $\log(2x^2+1)-\log(7x^2+8)=-\log 4$ but failed to apply the quotient rule of logarithms to solve for x. They wrote $\log(2x^2+1-7x^2+8)=-\log 4$ which is an incorrect approach. Moreover, other students wrote $\log 2x^2+\log 1+\log 4=\log 7x^2+\log 8$ which resulted to a wrong value of x. Extract 7.1 shows a sample of responses from a student who failed to answer this question correctly.

7. (a) Solve for n in the equation $16^{(3-n)} \times 2^{(1+n)} = \frac{1}{2}$. Leave the answer in improper fraction form.

Solution

$16^{(3-n)} \times 2^{(1+n)} = 1/2$

$1/2^{8(3-n)} \times 1/2^{n(1+n)} = 1/2$

$8(3-n) \times 2(1+n) = 1/2$

$24 - 8n \times 2 + 2n = 1/2$

$24 - 2 + 8n + 2n = 1/2$

$22 + 10n = 1/2$

$10n = 1/2 - 22/1$

$\frac{10n}{10} = \frac{22.5}{10} = 2\,25/100$

$n = 2\,25/100$ ∴ Improper fraction is $2\,25/100$

(b) Find the value of x in the equation $\log(2x^2 + 1) + \log 4 = \log(7x^2 + 8)$.

Solution

$\log(2x^2 + 1) + \log 4 = \log(7x^2 + 8)$

$\log(4x + 1) + \log 4 = \log 49x + 8)$

$(4x + 1) + 4 = 49x + 8$

$5 = 53x + 8$

$\frac{53x}{53} = \frac{13}{53}$

$\log x = \frac{13}{53}$

Extract 7.1: A sample of incorrect responses to question 7

In Extract 7.1, part (a), the student performed wrong calculations contrary to the laws of exponents and ended up with a wrong value of n. In part (b), the student cancelled the word "log" throughout the equation and hence added the terms contrary to the laws of logarithms.

Though the performance on this question was weak, there were students who managed to respond correctly to the question. The students were competent on the laws of exponents and logarithms. In part (a), they correctly solved the equation $16^{3-n} \times 2^{(1+n)} = \frac{1}{2}$ and obtained $n = \frac{14}{3}$ by using the laws of exponents. In part (b), they applied the laws of logarithms and methods of solving quadratic equations to obtain the value of x in the equation $\log(2x^2 + 1) + \log 4 = \log(7x^2 + 8)$. Lastly, they got $x = 2$ or $x = -2$, which was the correct answer. Extract 7.2 shows a sample of correct responses from one of the students who answered this question correctly.

7. (a) Solve for n in the equation $16^{(3-n)} \times 2^{(1+n)} = \frac{1}{2}$. Leave the answer in improper fraction form.

$16^{(3-n)} \times 2^{(1+n)} = 1/2$

$16^{(3-n)} \times 2^{(1+n)} = 2^{-1}$

$2^{4(3-n)} \times 2^{(1+n)} = 2^{-1}$

$2^{(12-4n)} \times 2^{(1+n)} = 2^{-1}$

$2^{(12-4n+1+n)} = 2^{-1}$

$2^{(13-3n)} = 2^{-1}$

$13-3n = -1$

$13+1 = 3n$

$\frac{14}{3} = \frac{3n}{3}$

$\therefore n = 14/3$

(b) Find the value of x in the equation $\log(2x^2+1)+\log 4=\log(7x^2+8)$.

$\log(2x^2+1)+\log 4=\log(7x^2+8)$

$\log 4=\log(7x^2+8)-\log(2x^2+1)$

From quotien law of logarithms,

$\log_a A-\log_a B=\log_a\left(\frac{A}{B}\right)$

$\log(7x^2+8)-\log(2x^2+1)=\log 4$

$\frac{\log\left(\frac{7x^2+8}{2x^2+1}\right)}{\log}=\frac{\log 4}{\log}$

$\frac{7x^2+8}{2x^2+1}=4$

$8x^2+4=7x^2+8$

$8x^2-7x^2+4-8=0$

$x^2-4=0$

Factorization By splitting the middle term,

$x^2+2x-2x-4=0$

$(x^2+2x)-(2x+4)=0$

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$x(x+2)-2(x+2)=0$

$(x-2)(x+2)=0$

$x=2$ OR $x={}^-2$

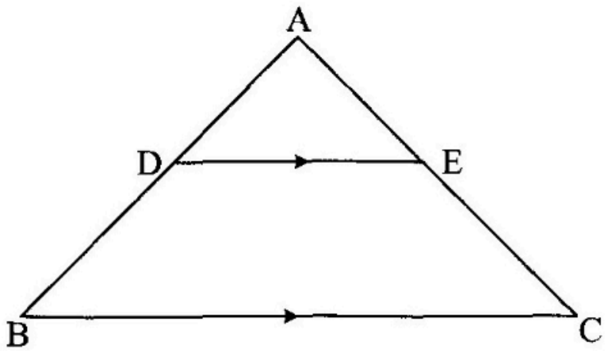
$\therefore$ The value of x is 2 or ${}^-2$

Extract 7.2: A sample of correct responses to question 7

In Extract 7.2, part (a), the student applied the laws of exponents correctly to obtain the value of n. In part (b), the student correctly applied the laws of exponents and logarithms to get the value of x.

2.8 Question 8: Congruence and Similarity

This question consisted of parts (a) and (b). Part (a) assessed the students' ability to find the length of $\overline{AE}$ in the following figure, where $\overline{AD}:\overline{BD}=\frac{3}{5}$ and $\overline{AC}=9.6$ cm.



In part (b), the students were required to prove that $A\hat{B}D = A\hat{C}D$, given that ABC is a triangle in which $\overline{AB} = \overline{AC}$ and D is the mid-point of $\overline{BC}$.

According to data, out of 695,143 students who attempted the question, 603,169 (86.8%) students scored below 3 marks. It was further noted that 438,606 (63.1%) students scored zero, indicating that the students' performance on this question was weak. In spite of weak performance, 9,149 (1.3%) students managed to score full marks for this question. Figure 8 summarizes the students' performance on question 8.

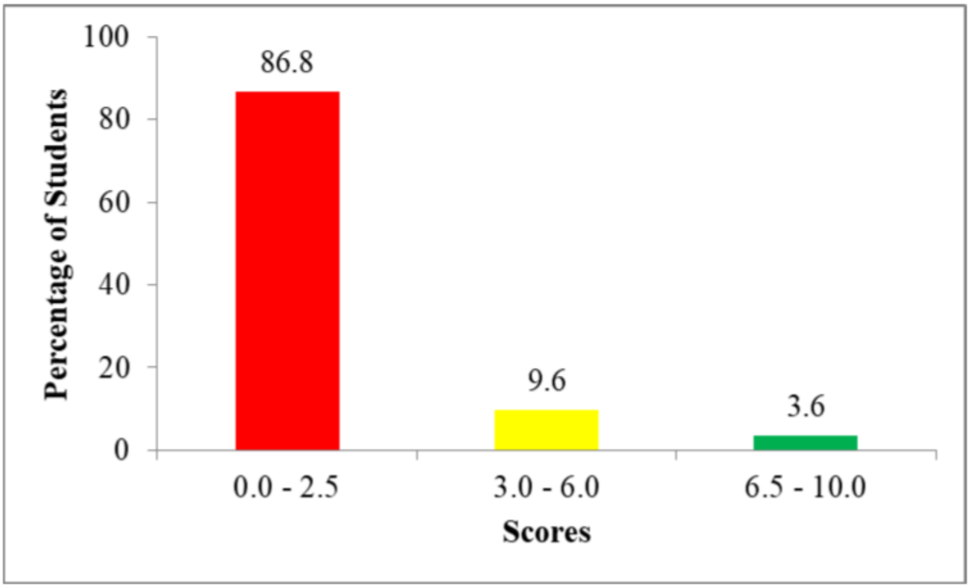
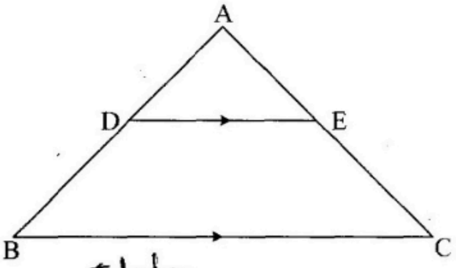


Figure 8: *Students' performance on question 8*

The analysis of students' responses indicates that the students who performed poorly on this question lacked knowledge and skills on the basic concepts of congruence and similarity. In part (a), the students were unable to recognize that $\overline{AD} = 3$ cm and $\overline{BD} = 5$ cm. Also, they failed to notice and make use of the relations $\overline{AE} + \overline{EC} = \overline{AC}$ and $\overline{BD} + \overline{AD} = \overline{AB}$. Other students failed to apply correctly the ratio theorem for similar triangles. They wrote $\overline{AD} : \overline{BD} = \overline{AE} : \overline{AC}$. Upon substituting the given measurements, they obtained $\overline{AE} = 5.76$ cm which was incorrect. Moreover, some students applied wrongly the Pythagoras' theorem to find $\overline{AE}$ as $\overline{AD}^2 + \overline{AC}^2 = \overline{AE}^2$ which resulted to incorrect answers. Furthermore, many students failed to recognize the two similar triangles from the given figure. In part (b), the students were not able to sketch the triangle ABC using the given information. This shows a lack of competence on the basic concepts of geometry. There were students who managed to sketch the triangle ABC but failed to give logical arguments to prove that $A\hat{B}D = A\hat{C}D$. Extract 8.1 shows a sample of responses from a student who answered this question incorrectly.

8. (a) In the given figure, $\overline{AD}:\overline{BD} = \frac{3}{5}$ and $\overline{AC} = 9.6$ cm. Find the length of $\overline{AE}$.



Solution

$\frac{\overline{AD}}{BD} = \frac{3}{5}$

$AC = 9.6cm$

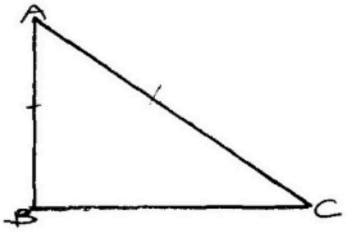
$9.6 \div 2$

$\frac{96}{10} \div 2$

$\frac{96}{10} \times \frac{1}{2} = \frac{48}{10}$

The length of $\overline{AE}$ is 4.8m

(b) ABC is a triangle in which $\overline{AB} = \overline{AC}$ and D is the mid-point of $\overline{BC}$. Prove that $A\hat{B}D = A\hat{C}D$.



$\overline{AB} = \overline{AC}$

$\overline{BC} = \overline{BA}$

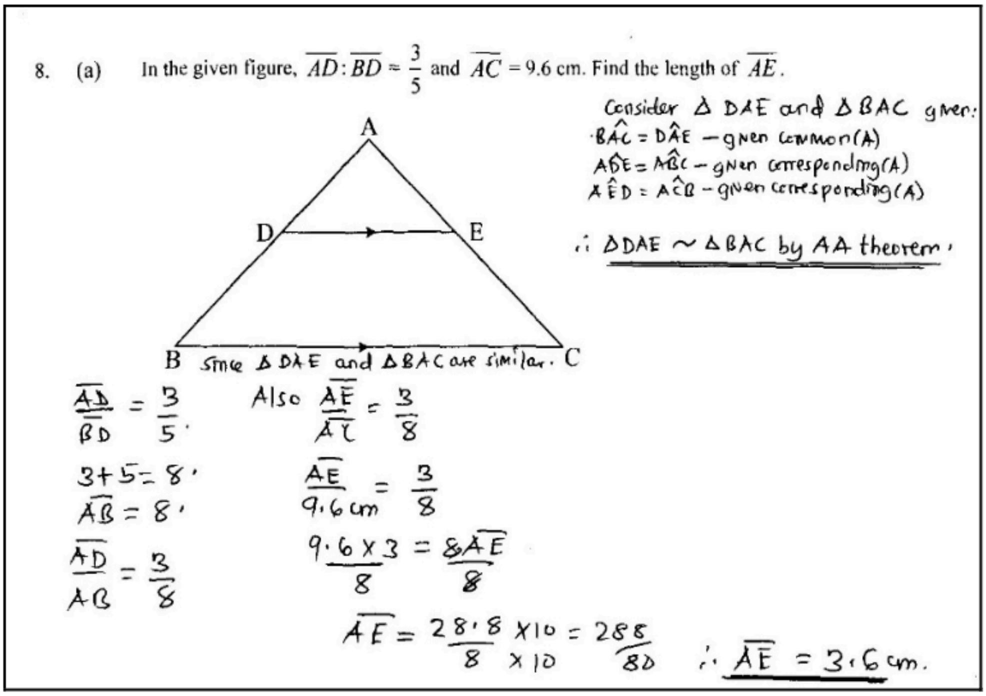
So $A\hat{B}D = A\hat{C}D$ by

$\frac{\overline{AB}}{\overline{BC}} = \frac{\overline{AC}}{\overline{BA}}$

Extract 8.1: A sample of incorrect responses to question 8

In Extract 8.1, part (a), the student failed to apply the ratio theorem in order to find the length of $\overline{AE}$. In part (b), the student failed to apply the theorems on congruence of triangles.

Although the performance was weak, the analysis of responses shows that there were students who performed well on this question. The students had adequate knowledge and skills in solving problems on congruence and similarity. In part (a), they were able to find the length of $\overline{AE}$ by using the given information $\overline{AD}:\overline{BD}=\frac{3}{5}$ and $\overline{AC}=9.6$ cm and then applied the ratio theorem for similar triangles to obtain $\overline{AE}=3.6$ cm. In part (b), the students gave logical arguments in proving that $A\hat{B}D=A\hat{C}D$. They systematically applied the properties and theorems of congruence. They either used the Side-Side-Side postulate (SSS), Side-Angle-Side (SAS) postulate or Right angle-Hypotenuse-Side (RHS) postulate. Extract 8.2 shows a sample of correct responses from a student who answered this question correctly.



8. (a) In the given figure, $\overline{AD}:\overline{BD}=\frac{3}{5}$ and $\overline{AC}=9.6$ cm. Find the length of $\overline{AE}$.

Consider ΔDAE and ΔBAC given:
$B\hat{A}C = D\hat{A}E$ – given common (A)
$A\hat{D}E = A\hat{B}C$ – given corresponding (A)
$A\hat{E}D = A\hat{C}B$ – given corresponding (A)

$\therefore \Delta DAE \sim \Delta BAC$ by AA theorem.

Since ΔDAE and ΔBAC are similar.

$\frac{\overline{AD}}{\overline{BD}} = \frac{3}{5}$

$3+5=8$

$\overline{AB}=8$

$\frac{\overline{AD}}{\overline{AB}}=\frac{3}{8}$

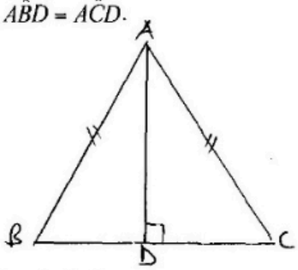
Also $\frac{\overline{AE}}{\overline{AC}}=\frac{3}{8}$

$\frac{\overline{AE}}{9.6\text{ cm}}=\frac{3}{8}$

$\frac{9.6\times 3}{8}=\frac{8\overline{AE}}{8}$

$\overline{AE}=\frac{28.8}{8}\frac{\times 10}{\times 10}=\frac{288}{80}$ $\therefore \overline{AE}=3.6$ cm.

(b) ABC is a triangle in which $\overline{AB} = \overline{AC}$ and D is the mid-point of $\overline{BC}$. Prove that $A\hat{B}D = A\hat{C}D$.



Consider ΔADB and ΔADC given

$\overline{AD}$ – given common (S)

$A\hat{D}B = A\hat{D}C$ – given right (A)

$\overline{AB} = \overline{AC}$ – given (S)

$\therefore \Delta ADB \cong \Delta ADC$ by SAS postulate.

But $A\hat{B}D = A\hat{C}D$ → corresponding angles

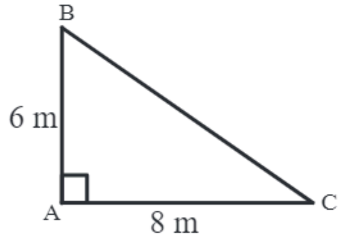
$\therefore A\hat{B}D = A\hat{C}D$ from the definition of congruence.

Extract 8.2: A sample of correct responses to question 8

In Extract 8.2, part (a), the student applied correctly the ratio theorem for similar triangles and obtained $\overline{AE} = 3.6$ cm. In part (b), the students responded correctly by applying Side-Angle-Side (SAS) postulate on congruence of triangles to prove that $A\hat{B}D = A\hat{C}D$.

2.9 Question 9: Trigonometry and Pythagoras' Theorem

The question consisted of two parts, (a) and (b). In part (a), the students were required to find the height of a building correct to one decimal place, given that the angle of elevation of the top of the building from a point on the ground is $25°$ and the point on the ground is 80 m from the base of the building. In part (b), the students were required to calculate the length of $\overline{BC}$ in the following figure.



The data analysis shows that the question was attempted by a total of 695,142 students and among them 232,222 (33.4%) scored 3 to 10 marks indicating that the students' performance on this question was average. Further analysis revealed that 404,323 (58.2%) students scored zero whereas 23,689 (3.4%) students scored all 10 marks. The summary of the students' performance on question 9 is shown in Figure 9.

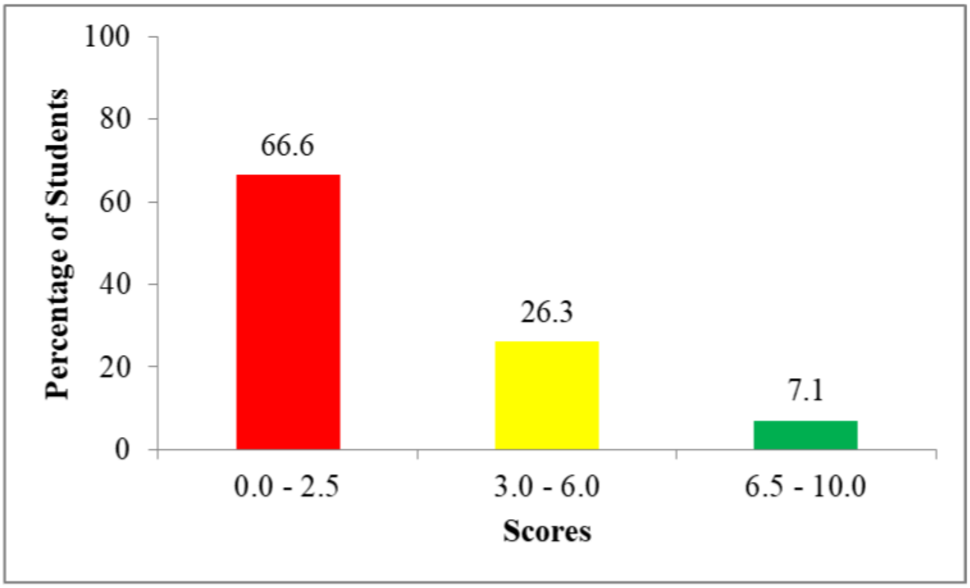
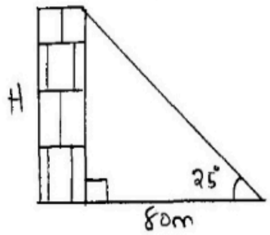


Figure 9: *Students' performance on question 9*

The analysis of responses shows that the students who performed well on this question were competent on the basic tenets of trigonometry and Pythagoras' theorem. In part (a), the students transformed the word problem into its equivalent geometrical form. They applied the

25° using the formula $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$, where θ is the angle of elevation of the building. They were able to substitute the given data in the formula, that is $\tan 25° = \frac{h}{80 \text{ m}}$, where h is the height of the building. They calculated the value of h to get $h = 37.3$ m. In part (b), the students recognized that the given figure is a right-angled triangle. Thus, they applied the Pythagoras' theorem to calculate the length of $\overline{\text{BC}}$ and correctly obtained $\overline{\text{BC}} = 10$ m. Extract 9.1 shows a sample of responses from a student who answered this question correctly.

9. (a) The angle of elevation of the top of a building from a point on the ground is 25°. If the point on the ground is 80 *m* from the base of the building, find the height of the building correct to one decimal place.



$\text{Tan}\,\theta° = \frac{\text{Opposite}}{\text{Adjacent}}$

$\text{Tan}\,25° = \frac{H}{80\text{m}}$

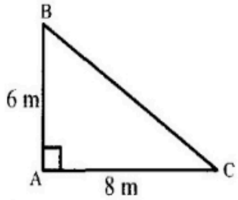
$80\text{m}\,(\text{Tan}\,25°) = H.$

$80\text{m}\,(0.4663) = H.$

$37.304\text{m} = H.$

$\therefore$ Height of the building is 37.3m

(b) Calculate the length of BC in the following Figure;



$\overline{AB} = 6m$

$\overline{AC} = 8\,m$

$\overline{BC} = ?$

using Pythagoras theorem

$a^2+b^2=c^2$

$\overline{AB}^2+\overline{AC}^2=\overline{BC}^2$

$6^2+8^2=\overline{BC}^2$

$36+64=\overline{BC}^2$

$\sqrt{100}=\sqrt{\overline{BC}^2}$

$\overline{BC}=10m$

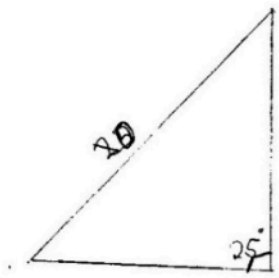
Extract 9.1: A sample of correct responses to question 9

In Extract 9.1, part (a), the student managed to find the height of the building after transforming the word problem into its geometrical form. In part (b), the student managed to determine the length of $\overline{\text{BC}}$ by using the Pythagoras' theorem.

On the other hand, the analysis of students' responses shows that the students who performed poorly on this question lacked the competence on the basic tenets of trigonometry and Pythagoras' theorem. In part (a), the students failed to transform the problem into geometrical form. They drew either a cuboid, trapezium, parallelogram or a square to illustrate the word problem which was an incorrect approach. Thus, they failed to recognize a suitable formula for calculating the height of the building. For example, they applied the formula for finding the area of a triangle, that is $A=\frac{1}{2}bh.$

$\overline{BC}$, $(\overline{BC})^2 = (\overline{AB})^2 + (\overline{AC})^2$. They added the lengths of the given two sides of a triangle, that is $6\,\text{m} + 8\,\text{m} = 14\,\text{m}$, which is an incorrect length of $\overline{BC}$. Furthermore, some students calculated the product of the lengths of the two sides of the triangle, that is $6\,\text{m} \times 8\,\text{m}$ to get $48\,\text{m}$ as the length of $\overline{BC}$. These responses indicate that the students lacked competence on trigonometry and application of Pythagoras' theorem. Extract 9.2 shows a sample of responses from a student who answered this question incorrectly.

9. (a) The angle of elevation of the top of a building from a point on the ground is 25°. If the point on the ground is 80 *m* from the base of the building, find the height of the building correct to one decimal place.



$a^2 + b^2 = c^2$

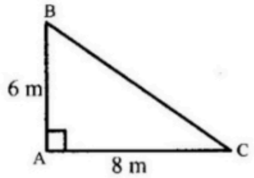
$80^2 + 25^2 = c^2$

$6400 + 625 = c^2$

$\sqrt{7025}$

$\therefore c^2 = \sqrt{7025}$

(b) Calculate the length of $\overline{BC}$ in the following Figure;



$a^2 + b^2 = c^2$

$6^2 + 8^2 = C^2$

$36 + 16 = C^2$

$C^2 = \sqrt{52}$

$\therefore$ The length of $\overline{BC}$ ~~tat~~ $11\sqrt{52}$

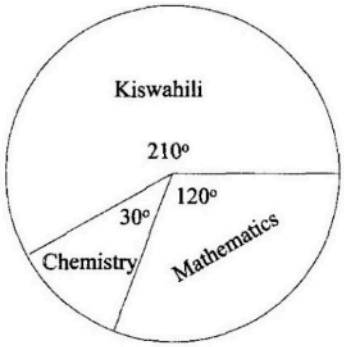
Extract 9.2: A sample of incorrect responses to question 9

In Extract 9.2, part (a), the student failed to transform the word problem into correct trigonometric forms and wrongly applied the Pythagoras' theorem instead of using the concepts on trigonometric ratios. In part (b), the student incorrectly substituted $8^2 = 16$ instead of $8^2 = 64$ in the equation $6^2 + 8^2 = c^2$ that led to incorrect length of $\overline{\text{BC}}$.

2.10 Question 10: Sets and Statistics

This question consisted of two parts, (a) and (b). In part (a), the students were required to find the total number of students in a certain school without using a Venn diagram. It was given that, 250 students attended on the first day of re-opening of the school, 350 students attended on the second day, 150 students attended on both first and second day, and 10 students were absent on both days, assuming that all registered students were supposed to attend the school on both days.

In part (b), the students were given the following pie chart which represents the number of students who passed a qualifying test in Mathematics, Chemistry and Kiswahili.



The students were assessed whether or not they are able to find; (i) the fraction of students who passed Kiswahili, (ii) the percentage of students who passed Mathematics, and (iii) the percentage of students who passed Mathematics and Chemistry.

The data analysis shows that the question was attempted by 695,121 students out of whom, 564,970 (81.3%) scored 0 to 2.5 marks. This reveals that the students' performance on this question was weak. However, 18,416 (2.6%) students scored full marks whereas 461,181 (66.3%) scored zero. The summary of the students' performance on question 10 is indicated in Figure 10.

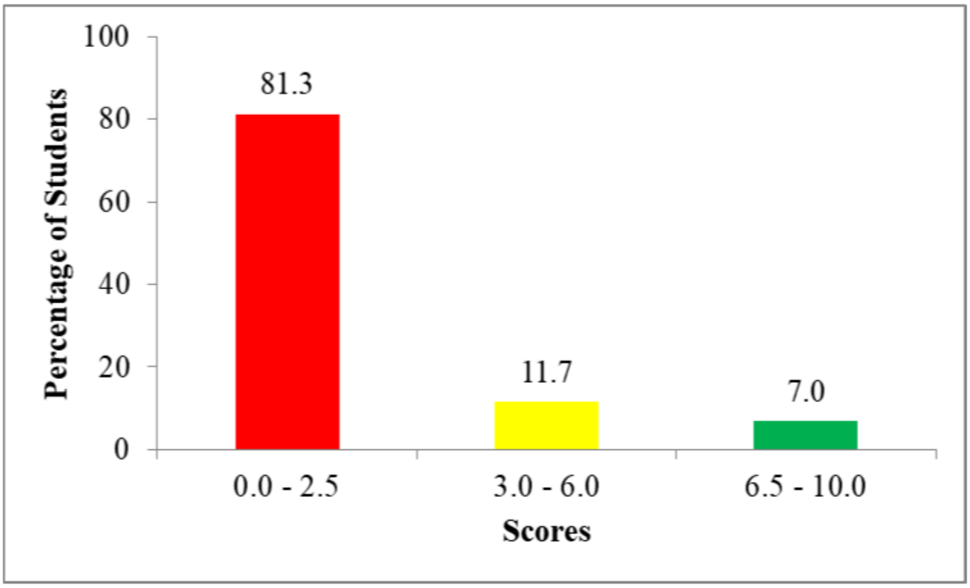
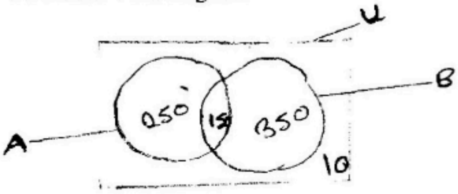


Figure 10: *Students' performance on question 10*

The analysis of responses shows that the students' weak performance on this question was contributed by lack of competence on the basic concepts of sets and statistics. In part (a), the students were not able to find the number of students registered at the particular school. They applied incorrect formulae such as $n(D_1 \cup D_2) = n(D_1 \cap D_2) - n(D_1)$ which led them to getting incorrect answers. In part (b), they failed to interpret and extract correct information from the pie chart. As a result, they were unable to obtain the fraction of students who passed Kiswahili. Similarly, the students were not able to compute the percentages as required. This highlights the lack of knowledge and computational skills on problems involving pie charts. Extract 10.1 shows a sample of incorrect responses from one of the students who failed to answer the question correctly.

10. (a) At a certain school, 250 students attended on the first day of re-opening of the school, 350 students attended on the second day and 150 students attended on both first and second day. It was further noted that 10 students were absent on both days. If all registered students were supposed to attend the school on both days, how many students does the school have? Do not use Venn diagram.



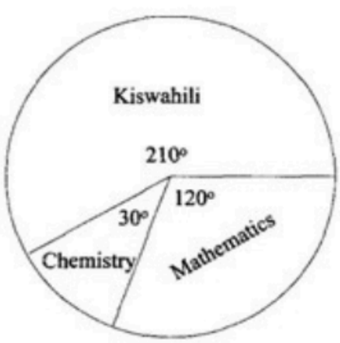
Soln

$250+350+150+x$

$x=600+150$

$x=750$

(b) The given Pie chart represents the number of students who passed a Qualifying Test in Mathematics, Chemistry and Kiswahili.



(i) the fraction of students who passed Kiswahili.

Soln

$120+30+210$

$=360$

$= 360 - 150 = 210$

Kiswahili: 210

(ii) the percentage of students who passed Mathematics.

Soln

210
+120
30
360

= 360
120
240

mathematics 240%

(iii) the percentage students who passed Mathematics and Chemistry.

Soln

120°+30°

=150

who student passed mathematics and chemistry is 150

Extract 10.1: A sample of incorrect responses to question 10

In Extract 10.1, part (a), the student tried to use a Venn diagram to solve the question which was contrary to requirement. In part (b)(i), the student failed to interpret the information in the pie chart. He/she just added the three degree measures and subtracted 150 from the sum. This was inappropriate computation. In part (b)(ii) and (iii), the student failed to calculate the required percentages. He/she made inappropriate calculations on the degree measures.

On the contrary, the analysis of responses shows that, the students who performed well on this question had sufficient competence on the basic concepts on sets and statistics. In part (a), they managed to determine the number of students registered in the school by appropriately applying the sets operation formulae $n(D_1 \cup D_2) = n(D_1) + n(D_2) - n(D_1 \cap D_2)$

$n(\mu) = n(D_1 \cup D_2) + n(D_1 \cup D_2)'$, D_1 is a set of students who attended school on the first day, D_2 is a set of students who attended school on the second day, and μ is a universal set, that is, the set of all students registered in the school.

In part (b)(i), they were able to interpret the information given in the pie chart, whereby they managed to find the fraction of students who passed Kiswahili. They divided the degree measure of students who passed Kiswahili by $360°$ to get $\frac{7}{12}$. In part (b)(ii), they calculated the percentage of students who passed Mathematics by dividing the degree measure representing the number of students who passed a Qualifying Test in Mathematics by $360°$ and then multiplied the result by 100% to get $33\frac{1}{3}\%$. In part (b)(iii), they first determined the sum of the degree measures of Mathematics and Chemistry and then divided it by 360°. Finally, they multiplied the resulting fraction by 100% to get $41\frac{2}{3}\%$. Extract 10.2 shows a sample of correct responses from one of the students who answered the question correctly.

10. (a) At a certain school, 250 students attended on the first day of re-opening of the school, 350 students attended on the second day and 150 students attended on both first and second day. It was further noted that 10 students were absent on both days. If all registered students were supposed to attend the school on both days, how many students does the school have? Do not use Venn diagram.

Solution

Let the number of students who attended on;
→ first day their set be A and
→ Second day their set be B.

Now; Given that. $n[A] = 250$

$n[B] = 350$

and $n[A \cap B] = 150$

Also $n[A \cap B]' = 10$

Then; from; $n[A \cup B] + n[A \cup B]' = n[U]$ ____ (i)

Again; But: $n[A \cup B] = n[A] + n[B] - n[A \cap B]$

$n[A \cup B] = 250 + 350 - 150$

$= 600 - 150$

$n[A \cup B] = 450$ ____ (ii)

Substitute eqn (ii) into eqn (i)

$n[A \cup B] + n[A \cup B]' = n[U]$

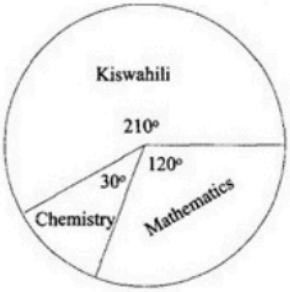
$450 + 10 = n[U]$

$n[U] = 460$

$n[U] = 460$ students.

∴ The school have 460 students.

(b) The given Pie chart represents the number of students who passed a Qualifying Test in Mathematics, Chemistry and Kiswahili.



Find:

(i) the fraction of students who passed Kiswahili.

Solution

Fraction of Kiswahili $= \frac{210^\circ}{360^\circ}$

$= \frac{21}{36}$

$= \frac{7}{12}$

$\therefore \frac{7}{12}$ passed kiswahili.

(ii) the percentage of students who passed Mathematics.

Solution

% Mathematics $= \frac{120^\circ}{360^\circ} \times 100\%$

$= \frac{1}{3} \times 100\%$

$= \frac{100\%}{3}$

$= 33\frac{1}{3}\%$

$\therefore 33\frac{1}{3}\%$ of students passed mathematics

(iii) the percentage students who passed Mathematics and Chemistry.

Solution

$$\% \text{ of Mathematics and Chemistry} = \frac{30^\circ + 120^\circ}{360} \times 100\%$$

$$= \frac{150^\circ}{360^\circ} \times 100\%$$

$$= \frac{15}{36} \times 100\%$$

$$= \frac{5}{12} \times 100\%$$

$$= \frac{500\%}{12} = \frac{125}{3}\%$$

$$= 41\tfrac{2}{3}\%$$

$\therefore 41\tfrac{2}{3}\%$ passed Math and Chemistry

Extract 10.2: A sample of correct responses to question 10

In Extract 10.2, part (a), the student used the correct formula to get the required number of students in the school. In part (b)(i), the student interpreted the pie chart correctly and calculated the fraction of students who passed Kiswahili. In part (b)(ii), he/she computed correctly the percentage of students who passed Mathematics. In part (b)(iii), the student computed the percentage of those who passed Mathematics and Chemistry after adding the degree measures, divided the result by 360° and then multiplied the fraction by 100%.

3.0 ANALYSIS OF STUDENTS' PERFORMANCE ON EACH TOPIC

The Basic Mathematics assessment paper consisted of 10 questions which were set from 20 topics. The analysis shows that only two (2) topics of *Trigonometry* and *Pythagoras' theorem* were rated average as 33.4 per cent of the students passed. The remaining topics were rated weak and the performance ranged from 27.6 to 9.4 per cent. The topics include: *Fractions* and *Decimals and Percentages* (27.6%), *Units* and *Ratios, Profit and Loss* (26.7%), *Geometry* and *Perimeters and Areas* (23.0%), *Coordinate Geometry* and *Geometrical Transformations* (22.4%), *Numbers* and *Approximations* (22.2%), *Sets* and *Statistics* (18.7%), *Algebra* and

Quadratic Equations (13.9%), *Congruence* and *Similarity* (13.2%) as well as *Exponents and Radicals* and *Logarithms* (9.4%). The Appendix on page 57 summarizes the students' performance on each topic.

4.0 CONCLUSION

The analysis reveals that 21.08 per cent of the students passed the assessment in 2023 Basic Mathematics assessment compared to 16.56 per cent of the students who passed the assessment in FTNA 2022. Based on the analysis, 33.4 per cent of the students had the highest performance in the topics of *Trigonometry* and *Pythagoras' theorem* while in 2022 Basic Mathematics assessment, 65.2 per cent had the highest performance in the topics of *Units* and *Approximations*. A total of 14 topics had constantly been poorly performed in both 2023 and 2022. Those topics are; *Decimals and Percentages*, *Ratios, Profit and Loss*, *Geometry*, *Perimeters and Areas*, *Coordinate Geometry*, *Geometrical Transformations*, *Numbers*, *Sets*, *Statistics*, *Algebra*, *Quadratic Equations*, *Congruence*, *Similarity*, and *Exponents and Radicals*.

5.0 RECOMMENDATIONS

The analysis shows that the students' performance was generally weak because 18 out of 20 topics had weak performance in 2023 Form Two National Assessment. Based on the findings, the following are recommended in order to improve the students' performance in future assessments:

(a) In the topic of Numbers, teachers should demonstrate to students on how to find the multiples of numbers by listing and identify the common multiples of the given two or more numbers by using relevant teaching resources like cards and charts of numbers.

(b) In the topic of Decimals and percentages, teachers should use teaching resources such as cents and shillings to guide students on the operation of decimals and percentages. Also, teachers should use the concepts taught in this topic to build a foundation for students to solve questions on rounding off, evaluation and estimations in real life.

(c) In the topic of Fractions, teachers should lead students to identify types of fractions and guide them in comparing and arranging fractions in order of magnitude by using teaching and learning resources like knives, pair of scissors, razor blades, manila papers, fruits, Cuisenaire rods.

(d) Teachers should demonstrate and guide students on how to convert one metric unit to another by using teaching resources like tape measure and charts of units of measurements in the topic of Units.

(e) In the topic of Ratios, Profit and Loss, teachers should demonstrate to students how to determine selling price, buying price, profit made, loss made, percentage profit and percentage loss by using relevant teaching and learning resources like money, worksheets and bank statements.

(f) Teachers should use various teaching and learning resources such as geoboard, rubber bands, triangles, quadrilaterals, polygonal shapes, ropes, tape measure and ruler to demonstrate to students on how to determine the angles, perimeters and areas by using appropriate formulae with real objects in the topic of Perimeters and Areas

(g) Teachers should guide students to apply algebraic operations techniques, including the BODMAS rule in computations of algebraic expressions in the topic of Algebra.

(h) In the topic of Quadratic equations, teachers should guide students through group discussions to solve quadratic equations step by step by using different methods, including the use of teaching and learning resources like manila charts, coloured chalks and marker pens.

(i) In the topic of Coordinate geometry, teachers should lead students in groups to discuss the meaning of gradient/slope and correct computations of the gradient/slope of a line joining two points by using teaching and learning resources like manila papers, geoboard, rubber bands and graph papers.

(j) Teachers should use teaching and learning resources like wall charts, number charts, multiplication tables and calculators to guide students to solve exponential and logarithmic equations and simplify algebraic expressions by using appropriate laws in the topics of Exponents and radicals, and Logarithms.

(k) Teachers should lead students' discussion on the conditions of similar and congruent figures and guide them to apply the conditions in computing lengths of unknown sides and proving for the similarity and congruence of figures. The use of teaching and learning resources like geometrical figures, photographs or pictures, geoboard, similar objects, mirrors and paper cuttings is important in attaining the intended competence in the topics of Congruence and Similarity.

(l) Teachers should use teaching and learning resources like geoboard, square cuttings, right-angled triangular objects, tape measure, rope, graph papers and clinometer to guide students to solve problems on the topics of Trigonometry and Pythagoras' theorem.

(m) In the topic of Sets, teachers should lead students on how to interpret mathematical problems in sets into mathematical languages and use relevant formulae to solve related problems. They should also lead students' discussion on how to interpret data from pie charts in the topic of Statistics. In teaching and learning these topics, various resources like playing cards, teams of players, protractors and pair of compasses can be used.

(n) During teaching and learning process, teachers are advised to design relevant hands-on activities to every topic and fully engage students. Teachers should also assess students regularly to determine the mastery of the intended competences.

Analysis of Students' Performance per Topic – FTNA 2023

S/N	**Topics**	**Question number**	**Percentage of students who scored 30 per cent or more**	**Remarks**
1	Trigonometry and Pythagoras' theorem	9	33.4	Average
2	Fractions, Decimals, and Percentages	2	27.6	Weak
3	Units, Ratio, Profit and Loss	3	26.7	Weak
4	Geometry, Perimeter, and Areas	4	23.0	Weak
5	Coordinate Geometry and Geometrical Transformations	6	22.4	Weak
6	Numbers and Approximations	1	22.2	Weak
7	Sets and Statistics	10	18.7	Weak
8	Algebra and Quadratic Equations	5	13.9	Weak
9	Congruence and Similarity	8	13.2	Weak
10	Exponents, Radicals, and Logarithms	7	9.4	Weak